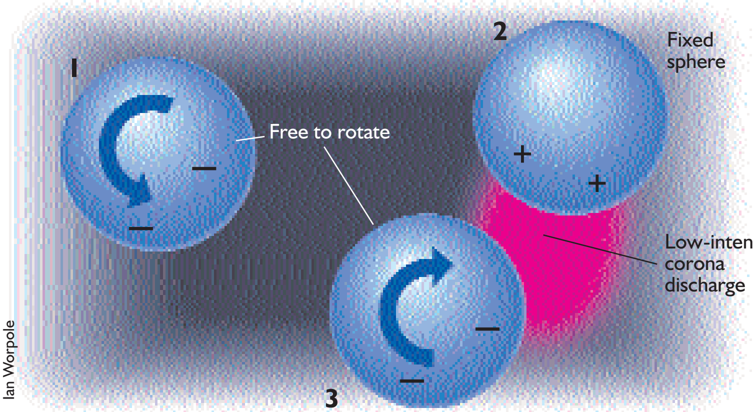
Spin and Energy—free?

The news brief published in August/September on pages 8–10 drew quite a few reader letters.

Your news brief reported on an experiment that appears to violate the law of conservation of energy and also questions the validity of electrostatics as we know it. Let me try to salvage both energy conservation and electrostatics by offering a more mundane mechanism for the phenomenon described (*1*). First, let us recall why, *in any configuration of external charges*, the static electric field cannot produce a torque required to spin a metal sphere about its center. The density of electrostatic force per unit of surface is $\sigma\mathbf{E}$, where σ is the surface charge density and $\mathbf{E}$ is the electric field. The density of torque, correspondingly, is $\sigma\,[\mathbf{E} \times \mathbf{R}]$. If there is no surface current, the electric field, $\mathbf{E}$, has only normal component parallel to the radius-vector $\mathbf{R}$, and, therefore, the density of torque is zero everywhere on the surface of the sphere. In order to have nonzero density of torque and, correspondingly, a possibility of nonzero net torque, there must be the surface current. Then, tangential to the surface component of the field $\mathbf{E}_t$, there may result the net torque $\boldsymbol{\tau}$, given by $\boldsymbol{\tau} = \int\sigma[\mathbf{E}_t \times \mathbf{R}]\, df$. Incidentally, the requirement that the constant torque must be accompanied by the persistent surface currents saves from extinction the energy conservation law. The spinning of an object requires a certain amount of electric power to be converted into mechanical work to compensate for inevitable friction. This is possible only when the current is drawn from the power source.



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What is the nature of current? Since the authors report that there was no sparking between the conductors, the only option is a low-intensity corona discharge. The potential difference of 400 to 5,000 V and a surface-to-surface separation of about 5 mm are sufficient for initiation of the corona. The mechanism of the appearance of the net torque in the three-spheres configuration is shown in the figure. Sphere 2 is fixed, and let us take it as positively charged. Spheres 1 and 3 are free to rotate, and both are negatively charged. The discharge is mainly confined between spheres 2 and 3 (red region). The presence of sphere 1 shifts the ionized cloud somewhat off the line connecting the centers of spheres 2 and 3. The surface current flowing out of the zone of discharge and the respective tangential component of the electric field create the net torque indicated by the arrow. Without sphere 1, the distribution of current and field would be symmetric and result in zero net torque. It would require too much hand-waving to try to explain the direction of rotation of sphere 1 on the basis of forces. It will suffice to invoke angular momentum conservation, since the angular momentum of the discharge current is negligible.

Finally, I want to emphasize strongly that, even though it is not immediately apparent from the treatment just given, the necessary condition for conversion of electric energy into mechanical energy is the hysteretic nature of the discharge. Namely, the discharge current can be maintained in the electric field lower than that required for its initiation. Further analysis of the three-sphere configuration is not particularly interesting because, apparently, this is an accidental arrangement and does not efficiently utilize the outlined effect.

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Reference

1. Wistrom, A. O.; Khachatourian, A. V. M. *Appl. Phys. Lett*. **2002**, *80*, 2800.

[*A. O. Wistrom replies*: According to the received view, the electrostatic force between conductors can be calculated in different ways. For example, it can be obtained by using the field method, where the electric field is first calculated by integrating over the whole charge distribution and then multiplying by the charge, or by using an action-at-a-distance approach, whereby the charge distribution is evaluated from Gauss's definition of surface potential, then summed over all charges according to Coulomb's law.

We (and others) find that both methods of analysis yield the same result for the electrostatic force between two spheres held at constant potential (*1*, *2*). However, this is not the case for three or more spherical conductors (*3*). Indeed, we find that the explicit solution to the generalized many-body electrostatic problem predicts the existence of a Coulomb torque notwithstanding the postulated direction of an auxiliary electric field. Rigorous theory predicts that electrostatic torque is the natural consequence of electrical action-at-a-distance force acting on an asymmetric distribution of charges residing on the surface of the conductors. The identification of electrostatic torque was prompted by experimental observations and has now been theoretically verified. Theory also shows that angular momentum is conserved, and hence the system obeys the force conservation laws. It remains to be seen whether an energy-producing device can be built. The technical challenges of constructing ideal conductors that are perfectly spherical are nontrivial.

The experimentally observed rotation cannot be explained using the conventional assumption of an electric field directed outward normal to an equipotential surface—an assumption that automatically precludes tangential forces. However, the observed rotation is correctly predicted by an explicit solution to the electrostatic problem, given Gauss's definition of boundary conditions on the conducting spheres and Coulomb's law of the electrostatic force, without invoking any approximations or simplifications with respect to symmetry or direction of an auxiliary electric field.

It is well to remember that the charge distribution on the spherical conductors is uniquely determined using Gauss's law of potentials. Experimentally, this means that, in isolation, a sphere held at constant potential (connected to a power supply) will have all its charges evenly distributed on its surface, and its corresponding center of charge will coincide with the sphere's center of mass. When a second conductor is brought into its vicinity, surface charges will instantaneously redistribute themselves under the action of their mutual influence, as a function of applied voltage and separation distance. We note that the charge distribution is axially symmetric because of the cylindrical geometry of two spheres. Also note that centers of charge no longer coincide with the centers of mass, even though they remain situated on the connecting axis.

Once a third conductor is introduced, the redistributed charges in this new configuration are no longer symmetrically distributed. Indeed, the charge distributions are now generally asymmetric on the sphere surfaces, with centers of charge that do not overlap the centers of mass. The reader can easily verify that force lines that originate from the centers of charge different from centers of mass are not generally perpendicular to the sphere's constant-potential surface, by drawing a set of lines extending outward from any position inside a circle but its center. Based on experimental observation and rigorous analysis, we conclude that the conventional assumption of an electric field perpendicular to an equipotential surface is inconsistent with Gauss's definition of surface potentials and Coulomb's law of the electrostatic force.

How is it possible that electrostatic torque has not been observed earlier, and why was the theory not worked out a long time ago, since it was over 200 years ago that Coulomb published his findings? A way to understand this apparent oversight is to study the history of electromagnetic discoveries and to learn about the important scientific and technological challenges at the time.

The first half of the 19th century was marked by a series of discoveries unveiling a variety of new phenomena in electricity and magnetism. The general task to which scientists and engineers then addressed themselves was to develop a unified theory of electromagnetism. For example, Thomson's initial work was in electrostatics, and he proposed a mathematical model prompted by certain analogies between electrostatics, as treated by Laplace and Poisson, and heat flow, as treated by Fourier.

The result was a mathematical approach that emphasized the spatial distribution and geometric relationships of electrical forces that could be expressed using the differential equations available at the time. Even though the concept of electrostatic potential was justified at the time for integrating electric and magnetic phenomena, the experimental verification of postulated electrical quantities, including the electrostatic force, received little or no attention. Indeed, the tremendous success of the field theory in solving important technological problems—such as design and operation of long signaling cables and, more recently, wireless communication devices—made it less important to evaluate its limitations.

So it is understandable that the analysis of electrostatic forces in many-body systems has had low priority during the last century. It was the burgeoning interest in nanotechnologies and the accompanying search for new methods of studying and manipulating particles one at a time that prompted our investigation of electrostatic forces in a many-body setting. The discovery of Coulomb torque was not expected. The identification of a rotational force—that in the absence of a restoring force could lead to spin—has far-reaching implications and invites investigation of systems of all size scales where the electrostatic force is the dominant operative force. This includes materials at the atomic and molecular scale and is relevant to understanding their spectral, electronic, and structural properties.]

References

1. Wistrom, A. O.; Khachatourian, A. V. Calibration of the electrostatic force. *Measurement Sci. Technol.* **1999**, *12* (10), 1296–1299.

2. Khachatourian, A. V. M.; Wistrom, A. O. Evaluation of the Coulomb force via the Fredholm integral equation. *J. Phys. A: Math. Gen.* **2000**, *33*, 307–317.

3. Khachatourian, A. V. M.; Wistrom, A. O. Coulomb torque—a general theory for electrostatic forces in many-body systems. *J. Phys. A: Math. Gen.* **2003**, *36*, 6495–6508; with corrections *36*, 8539.